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## Formal Semantics for the Progressive of Stative and Non-Stative Verbs

0.

In the first part of this contribution, we will present, as a starting point for the following discussions, a simple formal language  $P$  containing one stative<sup>1</sup> predicate. We will then discuss, on an intuitive level, how a treatment of predicates of change could be conceived, and how the progressive could be rendered in a formal language.

We will then give a formal definition of a language,  $TP_1$ , based on  $P$ , and we will construct a semantics for  $TP_1$ , which incorporates the ideas discussed.

### 1. *The language P*

Syntax:

Vocabulary: – the one-place predicate SLEEP;  
– infinitely many individual constants:  
JOHN, . . .

Well-formed

formulae:  $WFF_P$ , the set of well-formed formulae of  $P$ , is given by the following definition:  
– if  $X$  is an individual constant, then  $SLEEP(X) \in WFF_P$ ;  
– if  $S \in WFF_P$ ,  $S_1 \in WFF_P$ , then:  
a)  $\neg(S) \in WFF_P$ ;  
b)  $(S \wedge S_1) \in WFF_P$ ;  
– nothing else is an element of  $WFF_P$ .

Remark: Throughout this paper, the expressions of formal languages are used autonomously, i. e. as their own names.

\* For helpful comments, criticisms, discussions and encouragements, we are greatly indebted to LENNART ÅQVIST, MAX CRESSWELL, FRANZ GUENTHNER, HANS KAMP, and CHRISTIAN ROHRER. ALAN KIRKNESS kindly helped us with the English idiom. Preliminary versions of this paper were presented and discussed at a Symposium in Partizanske Vode, Yugoslavia, 2. VI. 1979, and we thank Prof. ALEXANDER KROHN, Belgrad, for his comments, and at a colloquium on 'Kontext und Vagtheit' at Bochum, 10.-14. III. 80

<sup>1</sup> In the terms of VENDLER 1967

The semantics for P is supplied by the following model M:

$\langle \{0, 1\}, D, SL, V \rangle$

The components of this model M are

- $\{0, 1\}$ , the set of (classical) truth values, where 0 stands for 'false' and 1 for 'true';
- D is a nonempty set, the individual domain;
- $SL \subseteq D$ ;
- V is a function, characterized by the following stipulations:
  - a) if X is an individual constant, then  $V(X) \in D$ ;
  - b)  $V(\text{SLEEP}) = SL$ ;
  - c) if X is an individual constant, then  $V(\text{SLEEP}(X)) = 1$  iff  $V(X) \in SL$ ;
  - d) if  $S \in \text{WFF}$ , then  $V(\neg(S)) = 1$  iff  $V(S) = 0$ ;
  - e) if  $S \in \text{WFF}$ , and  $S_1 \in \text{WFF}$ , then  $V(S \wedge S_1) = 1$  iff  $V(S) = V(S_1) = 1$ .

This language P, as we see, is a rather simplified fragment of predicate logic with the logical power of propositional logic. As problems of quantification are immaterial to our further discussion, we have no quantifiers and variables in our language P, and we only have one predicate, the (stative) SLEEP.

The semantics of P is classical in the treatment of both predicates and logical connectives.

2.

So the first question of real interest is: how could P be extended to express change, as do natural language expressions like '*to fall asleep*' or the corresponding German verb '*einschlafen*'? An expression of that sort designates a transition between two different states taking place at different periods of time. This leads us to a partial answer to our question: a formal language which contains such predicates of change should in its semantic part incorporate a 'flow of time'. More specifically, the valuation of expressions of such a language should depend on time.

This leads to the next question: should the evaluation of sentences be conceived as dependent on time-points or rather as dependent on time-intervals? Following Cresswell<sup>2</sup>, we accept the idea of evaluating sentences at intervals. By doing so, by the way, we are able to state an important difference between stative verbs like *sleep* and verbs (or verbal expressions) of change, like *fall asleep*: expressions of the first type are semantically uniform in the sense that from

<sup>2</sup> CRESSWELL 1977, in: ROHRER (ed.) 1977

(1) *John slept*

being evaluated as true at an interval  $\bar{t}$ , we can conclude for all subintervals  $\bar{t}'$  of  $\bar{t}$ , that (1) is evaluated as true at  $\bar{t}'$ , as well.

But the same does not hold for expressions of change: from

(2) *John fell asleep*

being evaluated as true at an interval  $\bar{t}$ , we can not draw the conclusion that (2) must be true for each subinterval of  $\bar{t}$ ; on the other hand, we can see that from (2) being evaluated as true at an interval  $\bar{t}$ , it follows that there must be at least one subinterval 'at the beginning' of  $\bar{t}$ , where

(3) *John did not sleep*

is evaluated as true. Evaluated at at least one other subinterval 'at the end' of  $\bar{t}$ , (1) must come out true.

Another and, it seems, more difficult problem is posed by sentences with non-stative verbs in the progressive: they have a sort of non-factual reading, they leave open whether the change is really carried out completely, whether the resulting state is ever reached or not, e.g.

*John was falling asleep when the phone rang.*

or

*The special agent was just capturing Hotzenplotz,  
when he was hit from behind with a club.*

where it is not understood that John really fell asleep or that the special agent captured Hotzenplotz. So, one approach to an analysis of the progressive of non-stative verbs would be to incorporate into our system something like the concept of a 'degree of realization' of a certain state of affairs, for we could then handle the progressive of non-statives as the increasing of the degree of realization of the resulting state of affairs, e.g. *falling asleep* would be treated as, roughly: become more and more asleep<sup>3</sup>.

But how can such an idea be built into formal semantics?

2.1.1.

A quite popular solution is to interpret the formal language by 'fuzzy semantics', where the extension of a predicate is no longer regarded to be a classical set of individuals but rather a 'fuzzy set', where the membership is 'graded'. Let for instance be FSL the fuzzy set replacing the classical set SL of 'sleepers' in our original model M. Then, formally FSL is characterized by a function FFSL from D into the closed interval of real numbers between 0 and 1, i.e. (0; 1), such that for each element d of D: FFSL (d) is the degree of

<sup>3</sup> CRESSWELL 1977, following DOWTY presents a different approach to the progressive in an intensional framework with a tree structure, where the actual world splits up, at one point, into different futures, one of them being picked later on out as the future of the 'real' world.

membership in FSL of  $d$ . Using this concept, we can define a fuzzy valuation function  $FV$  for the language  $P$  which assigns to each individual constant  $X$  an element of  $D$  and to each formula  $SLEEP(X)$  that real number  $r$  as truth value which is the degree of membership of  $FV(X)$  in  $FSL$ , i.e.:  $FV(SLEEP(X)) = FFSL(FV(X))$ .<sup>4</sup>

This looks quite appealing at first sight: sentences are not simply true or false, but true or false to a degree, and so we could, in principle, adopt fuzzy logics for our purpose of analyzing verbs of change and their progressives.

On second thoughts, however, we can see that the fuzzy logic approach is inadequate. To show that, we will try to give a complete 'fuzzy semantics' for our language  $P$ : a fuzzy model  $FM$  for  $P$  would be

$\langle (0; 1), D, FFSL, FV \rangle$

where

- $(0; 1)$  is the set of truth-values, i.e. the closed interval between the two 'classical truth values' 0 and 1, i.e. the set  $\{r \mid 0 \leq r \leq 1\}$ ;
- $D$  is as above;
- $FFSL$  is the 'characteristic function' of the 'fuzzy set'  $FSL$ , a function from  $D$  into  $(0; 1)$ ;
- $FV$  is the fuzzy valuation function, characterized as follows:
  - a) if  $X$  is an individual constant, then  $FV(X) \in D$ ;
  - b)  $FV(SLEEP) = FFSL$ ;
  - c) if  $X$  is an individual constant,  $FV(SLEEP) = FFSL(FV(X))$ ;
  - d)  $FV(\neg(S)) = 1 - FV(S)$ .

A short comment is in order: the proposed definition of the negation 'reverses' the truth-value of the embedded sentence, turns it round 0.5, as we might say. This is still not implausible.

- e) if  $S, S_1 \in WFF$ ,  
 $FV(S \wedge S_1) = ?$

To justify our putting the question mark here, we discuss some proposed or conceivable possibilities:

#### 2.1.1.1. *The minimum solution*

The minimum solution, which is the most popular one, would be

$$FV(S \wedge S_1) = \min(FS(S), FV(S_1))$$

This solution is in contradiction, we feel, to the intuitions of fuzzy logic, as we see in the following example:

Let  $S_0$  be  $S_1 \wedge S_2 \wedge S_3 \wedge S_4 \wedge S_5 \wedge S_6 \wedge S_7 \wedge S_8 \wedge S_9 \wedge S_{10}$

Let, relative to a model, hold:

$FV(S_1) = 1, FV(S_2) = 1, FV(S_3) = 1$  and so on to

$FV(S_9) = 1$ ; let  $FV(S_{10})$  be 0,

<sup>4</sup> See ZADEH 1965, and LAKOFF 1972

so  $S_0$  gets 0 as its value. But are we not frequently told, by the advocates of fuzzy logic, that classical logic is inadequate for the description of natural language for the very reason that natural language sentences may be true to a degree, and that a sentence must not be treated as totally false if some of the conditions for its truth hold and some don't? So, in our sentence  $S_0$  above, the conjunction of  $S_1$  to  $S_{10}$ , nine of the conditions given by the sentences  $S_1$  to  $S_9$  hold, and one does not, the one expressed by  $S_{10}$ , and still the standard interpretation of fuzzy logic treats the sentence as being totally false! So, in this case, fuzzy logic trades in the bivalence principle in return for more 'adequacy', but alas, from the standpoint of the fuzzifiers, there is no difference to classical logic. This point is frequently overlooked by advocates of fuzzy logic, as they invest much care in making their approach look good by discussing simple sentences, and much less care in justifying their treatment of the logical connectives.<sup>5</sup>

2.1.1.2. *The ratio solution*

One could try to correct what we found out above by giving the following semantics of  $\wedge$ :

$$FV(S_1 \wedge S_2 \wedge \dots \wedge S_n) = \frac{FV(S_1) + FV(S_2) + \dots + FV(S_n)}{n}$$

That would, in our example above, give 0.9 as the value of the conjunction, and that indeed seems more satisfactory from the standpoint of an advocate of fuzzy logics. But, on the other hand, in this treatment ( $S_1 \wedge \neg(S_1)$ ) gets 0.5 as value, regardless of the truth-value of both conjuncts. So before we . . . accept that solution, advocates of fuzzy logic should give some intuitive justification, as, from a classical standpoint, it is untenable that a contradiction is true to any degree.

Moreover, if we reduce  $(0; 1)$  to  $\{0, 1\}$ , this definition, unlike the minimum solution, still gives us 0.5 for  $(S_1 \wedge \neg(S_1))$ , and does not reduce to the classical definition of the connective  $\wedge$  in two-valued logic.<sup>6</sup>

2.1.1.3. *The multiplication solution*

Another conceivable solution would be

$$FV(S \wedge S_1) = FV(S) \cdot FV(S_1).$$

Here, the reduction of the interval  $(0; 1)$  to the set  $\{0, 1\}$  yields the 'classical' semantics of the connective. But it falls foul of the same problem as the minimum solution does, and, moreover,  $FV(S \wedge S) \neq FV(S)$ , if  $0 < FV(S) < 1$ , and it is not clear what this should mean intuitively.

So the fuzzy logic approach to the definition of the logical connections does not seem very promising, and, in addition, there is the serious question of how to justify the exact numerical values attributed to the simple sentences. The difficulties, we feel, arise when the concept of degree is given a logical status, because, insofar as the valuation of simple sentence is concerned, it is quite sensible that a concept like degree should be built into the semantic apparatus, but it should not be confused with the concept of a sentence 'being more or less true'.

2.1.2.

Another solution, due to Åqvist and Guentner, seems much more promising: the main idea of their approach is to introduce for each state of

<sup>5</sup> See, e.g. RIEGER 1976, and LAKOFF 1972

<sup>6</sup> See KAMP 1975, 130 ff, and TODT 1980

affairs  $U$  –  $U$  being an element of the power-set of  $T$  – a preference ordering  $\geq_u$  on the set of time-points  $T$ . Then  $t \geq_u t'$  is to mean that “time  $t$  realizes (or exhibits) the state of affairs  $U$  at least as much as time  $t'$ .”<sup>7</sup>

Letting  $U$  be the state of affairs corresponding to (4) *John sleeps*,

(5) *John falls asleep*

can be explained straightforwardly, its truth-conditions being rendered as

(5) is true at a time-interval  $\bar{t}$  iff

– first, there is a proper subinterval  $\bar{t}'$  at the beginning of  $\bar{t}$ , such that

(6) *John does not sleep*

is true at each element (time-point) of  $\bar{t}'$ , and for all points  $t, t'$  in  $\bar{t}'$ , if  $t$  is before  $t'$ , then  $t \geq_{U'} t'$  (where  $U'$  corresponds to *John does not sleep*), and

– second, there is a proper subinterval  $\bar{t}'' = \bar{t} \setminus \bar{t}'$ , such that *John sleeps* is true at each element of  $\bar{t}''$ , and for all  $t'', t'''$  in  $\bar{t}''$ , if  $t''$  is before  $t'''$ , then  $t'' \leq_U t'''$ .<sup>8</sup>

### 2.1.3.

Our aim is now to pass from the degree-of-realization-of-sentences-technique of Åqvist/Guenther to a treatment of predicates which allows in a formal way, to speak about their degree of realization. This would enable us to compare not only different time-points as to the realization of a certain sentence, but also different individuals at the same time-point with regard to a predicate.

To do so, we obviously have, in our model  $M$ , to replace the set  $SL$  we used for the interpretation of our predicate by a more apt construction.

The intuitive approach we take can be sketched informally as follows: a part of the semantic competence of a speaker of a language, concerning the semantics of predicating words, is that, confronted with a predicate, he can not only say – in principle – of each individual he is presented whether or not it ‘falls under the predicate’, but that he can also compare different individuals and recognize which of them is a more typical species. For this latter ability, it is immaterial whether the individuals in question are in the extension of a predicate or not: even if a speaker of English knows e.g. that neither Moby Dick nor Ahab are fish, he will still be in a position to agree that Moby Dick is more of a fish than Ahab is.

This can be reconstructed formally by introducing a two-place-relation  $\geq_{\text{FISH}}$  on the domain of individuals  $D$  which is reflexive, transitive and connected.  $d \geq_{\text{FISH}} d'$  means that the individual  $d$  is at least as much of a fish as the individual  $d'$ . Let  $=_{\text{FISH}}$  be that two place relation on  $D$  such that  $d =_{\text{FISH}} d'$  iff  $d \geq_{\text{FISH}} d'$  and  $d' \geq_{\text{FISH}} d$ . Obviously  $=_{\text{FISH}}$  is reflexive, transitive,

<sup>7</sup> ÅQVIST/GUENTHNER 1978, 171

<sup>8</sup> See ÅQVIST/GUENTHNER 1978, Chapter 7

and symmetrical, hence an equivalence relation. Thus  $=_{\text{FISH}}$  defines a partition of  $D$  into equivalence-classes, each of which contains just those individuals having the same 'degree of fishness'. We designate the set of these equivalence-classes by " $\Sigma_{\text{FISH}}$ ".

Using then the original relation  $\geq_{\text{FISH}}$ , a strict ordering  $>_{\text{FISH}}$  on  $\Sigma_{\text{FISH}}$  can be defined such that for all  $\delta_{\text{FISH}}, \delta'_{\text{FISH}} \in \Sigma_{\text{FISH}}$  and all  $d \in \delta_{\text{FISH}}, d' \in \delta'_{\text{FISH}}$  it holds:

$$\delta_{\text{FISH}} >_{\text{FISH}} \delta'_{\text{FISH}} \text{ iff } d \geq_{\text{FISH}} d' \text{ and } d \neq_{\text{FISH}} d'.$$

For this reason the ordered pair  $\langle \Sigma_{\text{FISH}}, >_{\text{FISH}} \rangle$  can be regarded as a scale (more specifically: an ordinal scale) where each element  $\delta_{\text{FISH}}$  of  $\Sigma_{\text{FISH}}$  is a 'degree of realization' of the predicate FISH. An appropriate measurement of individuals with respect to their degree of realization of the predicate is accomplished by defining a measurement-function  $\mu_{\text{FISH}}$  as follows:

$\mu_{\text{FISH}}$  is that function from  $D$  onto  $\Sigma_{\text{FISH}}$

for which for all  $d \in D, \delta_{\text{FISH}} \in \Sigma_{\text{FISH}}$ :

$$\mu_{\text{FISH}}(d) = \delta_{\text{FISH}} \text{ iff } d \in \delta_{\text{FISH}}.$$

Now  $\mu_{\text{FISH}}$  provides for each individual  $d$  its 'degree of fishness' as the value of  $\mu_{\text{FISH}}$  applied to  $d$ . The fact that a given  $d$  is a fish, then, can be established by postulating that  $\mu_{\text{FISH}}(d)$  must be equal to or greater than a given minimal degree of fishness. Therefore, one of the values of  $\mu_{\text{FISH}}$ , i. e. of elements of  $\Sigma_{\text{FISH}}$ , has to be designated as the threshold-value  $\tau_{\text{FISH}}$ .

We recognize that the assumption that there is exactly one such threshold-value surely oversimplifies the facts. Some of the problems connected with thresholds are discussed elsewhere.<sup>9</sup>

Given all this, the extension of the predicate FISH could be defined as  $\{d/\mu_{\text{FISH}}(d) >_{\text{FISH}} \tau_{\text{FISH}} \text{ or } \mu_{\text{FISH}}(d) = \tau_{\text{FISH}}\}$ .

Then the extension of any predicate of a given predicate language can be defined in an analogous way, with the slight complication that the concepts developed so far for the measurement of individuals with respect to one-place-predicates have to be extended for the measurement of  $n$ -tuples of individuals with respect to  $n$ -place predicates.

It is important to notice, by the way, that in this conception no predicate-independent numerical degrees of realization (as in fuzzy-semantics) are used. Moreover, for each predicate there are exactly as many degrees as there are differentiations between individuals with respect to that predicate. So the above mentioned difficulty of fuzzy-logics to justify intuitively the degree of membership in terms of a *real number* does not arise.

Going now back to our language  $P$ , it is obvious how the ideas developed so far can be used for the semantic interpretation of the predicate SLEEP. First, we have to replace the set  $SL$  in the model  $M$  by the ordered quadruple  $\langle \Sigma_{\text{SLEEP}}, >_{\text{SLEEP}}, \mu_{\text{SLEEP}}, \tau_{\text{SLEEP}} \rangle$ , which defines a measurement for the degree

<sup>9</sup> For a discussion of the ideas involved, see FROSCH 1980, and BALLWEG/FROSCH 1979.

of sleeping of individuals. Secondly, the original evaluation-function  $V$  has to be replaced by a function  $V'$  which is defined like  $V$  with the exception that the clauses b) and c) in its definition are replaced by: if  $X$  is an individual constant, then  $V'(\text{SLEEP}(X)) = 1$  iff  $\mu_{\text{SLEEP}}(V'(X)) \geq \tau_{\text{SLEEP}}$ .

This, of course, is not yet an important change, insofar as the expressive power of  $D$  is not increased. Especially, the connectives of  $P$  get the classical interpretation.

Let us now turn to the question of how to incorporate time into our semantic concepts. Basically, we notice that the degree of realization of SLEEP by a given individual may vary with time. Therefore, the elements of  $\Sigma$  (we omit the indices "SLEEP" in the sequel, since SLEEP is the only predicate of  $P$ ) can no longer be regarded as equivalence-classes of individuals, but rather as equivalence-classes of ordered pairs of individuals and time-points. For the same reason the measurement-function must be such that it assigns potentially different values to the same individual at different time-points. So we must either have a two-place function taking individuals and time-points, or, equivalently, a separate one-place function for each time-point. Finally we have to allow for different threshold-values for different time-points.

So the measurement  $\langle \Sigma, >, \mu, \tau \rangle$  has to be replaced by a time-dependent measurement which is an ordered triple  $\langle \Sigma, >, \langle \mu_t, \tau_t \rangle_{t \in T} \rangle$ , such that  $\Sigma$  is a set as described above,  $>$  is a strict ordering on  $\Sigma$ ,  $\langle \mu_t, \tau_t \rangle_{t \in T}$  is a family of ordered pairs of measurement functions and threshold values, indexed by the set  $T$  of time-points. An exact formulation of all this could be given along the same lines as for time-independent measurement, but starting with an original relation between ordered pairs of individuals and time-points. We only mention that for each  $d \in D$ ,  $\delta \in \Sigma$ , and  $t \in T$ , the function  $\mu_t$  has to be defined such that  $\mu_t(d) = \delta$  iff  $\langle d, t \rangle \in \delta$ .

### 3. The language TP 1

We are now in a position to set up a language TP 1 containing  $P$ , in the frame of which we will give an analysis of a predicate of change.

Syntax of  $TP_1$ :

Vocabulary: All the vocabulary of  $P$ , and the operators CA, CINGA, and ING, intuitively read as come about, coming about and progressive (for statives) respectively.

Well-formed

formulae:  $WFF_{TP_1}$  the set of well-formed formulae of  $TP_1$ , is given by the following definition:

- If  $X$  is an individual constant, then  $\text{SLEEP}(X) \in WFF_{TP_1}$ ;
- If  $X$  is an individual constant, then  $\text{CA}(\text{SLEEP}, X)$ ,  $\text{CINGA}(\text{SLEEP}, X)$  and  $\text{ING}(\text{SLEEP}, X)$  are in  $WFF_{TP_1}$ ;

- If  $S \in \text{WFF}_{\text{TP1}}$ , then  $\neg(S) \in \text{WFF}_{\text{TP1}}$ ;
  - If  $S_1, S_2 \in \text{WFF}_{\text{TP1}}$ , then  $(S_1 \wedge S_2) \in \text{WFF}_{\text{TP1}}$ .
- Nothing else is in  $\text{WFF}_{\text{TP1}}$ .

### Semantics of TP1:

A model TM1 for TP1 is a quintuple

$\langle \{0, 1\}, D, \langle \Sigma, >, \langle \mu_t, \tau_t \rangle_{t \in T} \rangle$ ,  
 $\langle \langle T, < \rangle, \bar{T}, \bar{t}_0 \rangle, \sqrt{\quad} \rangle$ , where

- $\{0, 1\}, D$  are as in M;
- $\langle \Sigma, >, \langle \mu_t, \tau_t \rangle_{t \in T} \rangle$   
 is a scalar measurement for the ‘degree of sleeping’,  
 replacing the set SL of our original model M, where:
  - a)  $\Sigma$  is a set of ‘values’, of ‘degrees of sleeping’, founded  
 on the ideas discussed above, and  $>$  is an irreflexive,  
 transitive and connected ordering on  $\Sigma$ ; so  $\langle \Sigma, > \rangle$   
 together form an ordinal scale;
  - b)  $\langle \mu_t, \tau_t \rangle_{t \in T}$  is a family of ordered pairs, indexed by T,  
 such that for all  $t \in T$   
 $\mu_t$  is a function from D onto  $\Sigma$ ,  
 $\tau_t$  is a designated element of  $\Sigma$ ;
- $\langle \langle T, < \rangle, \bar{T}, \bar{t}_0 \rangle$   
 is a temporal frame, where:
  - a)  $\langle T, < \rangle$  is the ‘flow of time’, where:  
 $T$  is a set of ‘time-points’;  
 $<$  is an irreflexive, transitive, dense and connected  
 ordering on T, such that for all t in T,  $\exists t' : t' < t$ ,  
 and  $\exists t'' : t < t''$ ;
  - b)  $\bar{T}$  is the set of time-intervals, which is to be that subset  
 of  $\mathcal{P}(T)$ , the power-set of T, such that for all  $\bar{t} \in \bar{T}$ ,  
 for all  $t, t', t'' \in T$ , if  $t \in \bar{t}$  and  $t' \in \bar{t}$ , and  $t < t'' < t'$ ,  
 then  $t'' \in \bar{t}$ .
  - c)  $\bar{t}_0$  is a designated element of  $\bar{T}$ , the ‘speech-interval’;

Remark:

By  $(t_0; t_1)$  we understand the set

$\{t \mid t \in T \text{ and } t_0 \leq t \leq t_1\}$ , i. e. the closed interval determined by  $t_0$  and  $t_1$ .

- $\sqrt{\quad}$  is a function such that for all individual constants X,  
 all  $\bar{t} \in \bar{T}$ , and all  $S, S_1, S_2 \in \text{WFF}_{\text{TP1}}$ :
  - a)  $\sqrt{\quad}(X, \bar{t}) \in D$ , and for all  $\bar{t}' \in \bar{T}$ ,  $\sqrt{\quad}(X, \bar{t}) =$   
 $\sqrt{\quad}(X, \bar{t}')$ ;
  - b)  $\sqrt{\quad}(\text{SLEEP}(X), \bar{t}) = 1$  iff, for all  $t \in \bar{t}$ ,  $\mu_t(V(X)) \geq \tau_t$ ;
  - c)  $\sqrt{\quad}(\text{ING}(\text{SLEEP}, X), \bar{t}) = 1$  iff there is an interval  
 $(t_1; t_2)$ , such that  $\bar{t} \subset (t_1; t_2)$ ,  
 and for all  $t_3 \in (t_1; t_2)$ ,  $\mu_{t_3}(V(X)) \geq \tau_{t_3}$ ;

- d)  $\sqrt{\text{CA(SLEEP, X), } \bar{t}} = 1$  iff  $\bar{t} = (t_1; t_2)$ , and there is a  $t_3 \in \bar{t}$  such that  $t_1 < t_3 < t_2$ , and for all  $t_4 \in \bar{t}$ , if  $t_4 < t_3$ , then  $\mu_{t_4}(\sqrt{X}) < \tau_{t_3}$ , and if  $t_3 < t_4$ , then  $\mu_{t_4}(\sqrt{X}) \geq \tau_{t_3}$  where  $\tau_t$  is fixed throughout  $\bar{t}$ , i.e. for all  $t, t'$  in  $\bar{t}$ :  $\tau_t = \tau_{t'}$ ;
- e)  $\sqrt{\text{CINGA(SLEEP, X), } \bar{t}} = 1$  iff  $\bar{t} = (t_1; t_2)$  and  $\mu_{t_1}(\sqrt{X}) < \mu_{t_2}(\sqrt{X})$ , and for all  $t_3 \in \bar{t}$ ,  $\mu_{t_3}(\sqrt{X}) < \tau_{t_3}$  (where  $\tau_t$  is again fixed throughout  $\bar{t}$ );
- f)  $\sqrt{\neg(S), \bar{t}} = 1$  iff  $\sqrt{S, \bar{t}} = 0$ ;
- g)  $\sqrt{((S_1 \wedge S_2), \bar{t})} = 1$  iff  $\sqrt{S_1, \bar{t}} = \sqrt{S_2, \bar{t}} = 1$ <sup>10</sup>. If  $S$  is a well-formed formula of TP 1, then  $S$  is said to hold at  $\bar{t}$  iff  $\sqrt{S, \bar{t}} = 1$ , and  $S$  is said to be true in TM 1 iff  $\sqrt{S, \bar{t}_0} = 1$ .

Remarks:

Clause a) guarantees that individual constants do denote rigidly. b) sets that a formula SLEEP(X) holds at an interval if the value of  $\mu_t$  for  $\sqrt{X}$  is at least at the threshold level for all time-points  $t$  in that period. We note, by the way, that this formulation contains in the nutshell the possibility of accounting for comparative constructions, like in: John sleeps deeper than Bill.<sup>11</sup> c) stipulates for the progressive of statives, that a sentence in the progressive, roughly speaking, holds at an interval if this interval is, in turn, a subinterval of another at which a sentence in the simple form would hold.<sup>12</sup> d) stipulates, simplifying somewhat, that a formula (CA(SLEEP, X) holds at an interval  $(t_1; t_2)$ , if this interval is 'split' by a point  $t_3$ , such that SLEEP(X) holds at  $(t_3; t_2)$ , and does not hold at  $(t_1; t_2) \setminus (t_3; t_2)$ .<sup>13</sup> e) the clause for the progressive of verbs of change is somewhat more complex: unlike in the case of ING, we can do here just with one interval  $(t_1; t_2)$ , and CINGA(SLEEP, X) holds at such an interval, iff SLEEP(X) does not hold at it, and, in addition, the individual denoted by X 'moves' toward the threshold value in  $(t_1; t_2)$ .<sup>14</sup>

<sup>10</sup> Another solution for the connective  $\wedge$  is discussed in CRESSWELL 1977.

<sup>11</sup> Cf. BALLWEG/FROSCH 1979

<sup>12</sup> This is parallel to the proposal in ÅQVIST 1977

<sup>13</sup> This is a more polished version of the definition proposed in BALLWEG/FROSCH 1979; again, there is a parallel in ÅQVIST 1977.

<sup>14</sup> The possibility of accounting for the progressive in this framework was called to our attention by MAX CRESSWELL at the Konstanz Colloquium, September 1978.

There is one problem with this solution as it stand: it would render "John is falling asleep." true in the case where John just changes from being wide awake to being awake to a lower degree. This was called to our attention by Peter Bosch at the Bochum conference. A solution would be possible in our framework by replacing  $\tau_t$ , for all  $t$ , by the pair  $(\tau_t^+, \tau_t^-)$ , where the new evaluation function  $\sqrt{\text{SLEEP}(x), \bar{t}}$  would be 1 iff  $\mu_t(\sqrt{x}) \geq \tau_t^+$ , for all  $t \in \bar{t}$ ,  $\sqrt{\text{SLEEP}(x), \bar{t}}$  would be 0 iff  $\mu_t(\sqrt{x}) \leq \tau_t^-$ , and undefined otherwise.

$\sqrt{\text{CINGA(SLEEP, x), } \bar{t}}$  would be 1 iff  $\bar{t} = (t_1; t_2)$ , and  $\mu_{t_1}(\sqrt{x}) \leq \mu_{t_2}(\sqrt{x})$ , and for all  $t_3 \in \bar{t}$ ,  $\tau_{t_3}^- < \mu_{t_3}(\sqrt{x}) < \tau_{t_3}^+$  where  $\tau$  is fixed throughout  $\bar{t}$ , and 0 otherwise.

We only mention this possibility here, because its working out would require either a three-valued logic in the spirit of U. Blau, or else a supervaluation approach. This latter one will be given in another paper of ours.

f) and g) are as usual; for g), a more sophisticated and satisfactory solution in an interval semantics is to be found in Cresswell 77.<sup>15</sup>

The truth definition simply says, that a formula S of TP 1 is true in a model iff it holds at the designated interval.

Our solution for the treatment of the progressive of expressions of change is in accordance to our intuition that a simple sentence in the progressive with a verb of change does not imply that the change be really carried through. Moreover, we have reached this result without introducing possible worlds, like the solutions mentioned in footnote 3.

We already pointed out above that the Åqvist/Guenthner – approach<sup>16</sup> influenced our own ideas very much; the only important difference is, that we do not take the concept of, ‘degree-of-realization-of-sentences’ as basic, but try to give an account of the ‘degree-of-realization-of-predicates’ by the construction of our scalar measurement, founded on the intuitions discussed in 2.1.3. This seems an advantage, as a treatment of comparison phenomena in formal semantics will urge the need for such a construction anyway. In addition we can point to the possibility of accounting for phenomena of vagueness and context-dependency within such a semantics based on scales.<sup>17</sup> This seems to indicate that the scales we use are indeed more than an ad-hoc device for the solution of the problems of verbs of change.

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<sup>15</sup> CRESWELL 1977

<sup>16</sup> ÅQVIST/GUENTHNER 1978, 178

<sup>17</sup> For a discussion, see FROSCH 80 and TODT 80

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